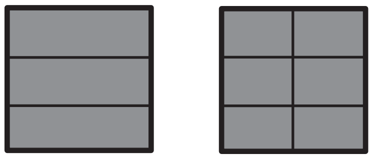
Mr. Brown the baker wants us to behold a brilliant fact about the beautiful brownies we've been working with. Here are two pans filled with his beautiful brownies. The first pan has been divided into thirds and the second pan has been divided into sixths.



Your teacher will give you some paper brownies and pans. You can see from the picture that one piece of the brownies from the first pan is the same amount as two pieces from the second pan.

Go ahead and take two of the smaller brownies, the ones that show the sixths and lay them on top of one of the pieces in the first pan that shows a third to see for sure if they match. 

Now let's show this same relationship using fractions. What fraction do we use to show one of the pieces from the pan divided into thirds?

Yes, $^1/_3$. What fraction do we use to show the two pieces from the pan divided into sixths?

Yes, $^2/_6$. From looking at the paper brownies we know these two fractions show the same amount for this size brownie pan. We can set up a math sentence to show this relationship: $^1/_3 = {}^2/_6$. We say that these two fractions are equivalent because they show the same ratio or relationship between the pieces of brownies and the whole pans. We introduced equivalent fractions in the previous level. But let's make sure you remember how this works. **In order for two fractions to be equivalent, they must show the same relationship between the parts to the whole.**

Looking at the two pans in front of you and the pictures of the pans above, we can see that the second pan has been divided into twice as many pieces as the first pan. We can show this as another math sentence. When we multiply the denominator in the first fraction by 2 we get six. For the fractions to show the same ratio or relationship, we have to be able to multiply the numerator in the first fraction by 2 and get the numerator given in the second fraction. We know from the multiplication facts that 1 x 2 = 2.

$$\frac{1}{3} \quad \text{x} \quad \frac{?}{2} \quad = \quad \frac{2}{6}$$

The complete math sentence looks like this. What do you notice about the middle fraction? What does it stand for?

$$\frac{1}{3} \quad \text{x} \quad \frac{2}{2} \quad = \quad \frac{2}{6}$$

Yes, 1. Any fraction that has the same numeral in the numerator and denominator shows 1 whole. This means that when we multiplied $^1/_3$ by that fraction, what we were really doing was multiplying it by 1. The Identity Property of Multiplication tells us that any number times 1 is that number—the value of that number remains the same. That's what makes $^1/_3$ and $^2/_6$ equivalent fractions.

We can use this property of multiplication to find other equivalent fractions. Let's keep going with $^1/_3$. Does it have an equivalent fraction using ninths?

$$\frac{1}{3} \quad \text{x} \quad \frac{?}{?} \quad = \quad \frac{?}{9}$$

Look at the denominator first. What number times 3 equals 9? Or to say it another way, does 3 go into 9 evenly?

Yes, 3 times. So we put the 3 into the denominator of the middle fraction. We also put it in the numerator because we want that fraction to equal 1.

$$\frac{1}{3} \quad x \quad \frac{3}{3} \quad = \quad \frac{?}{9}$$

Now we can find the numerator of the equivalent fraction in ninths by multiplying. 1 x 3 = 3. This means $^1/_3$ and $^3/_9$ are equivalent fractions.

We can check our work using the brownies. Start with the pan of brownies that shows the thirds. Put one of the $^1/_3$ brownies on the pan. Fill the pan divided into ninths with the $^1/_9$ brownies. Then take three of those and put them on top of the $^1/_3$ brownie. Do they line up?

You also can use the Identity Property along with the multiplication facts to find any number of equivalent fractions. For example, *find three equivalent fractions for $^1/_2$.*

We start with the known fraction. We know that we can multiply it by any fraction that shows one whole —where the numerator and denominator are the same—and the fraction we get will be equivalent to $^1/_2$. In this problem, we can choose any three equivalent fractions, so it is easiest to start with the first few math facts for that number. We then multiply the digits in the numerators and multiply the digits in the denominators to find the equivalent fractions.

$$\frac{1}{2} \text{ x } \frac{2}{2} = \mathbf{\frac{2}{4}} \qquad \frac{1}{2} \text{ x } \frac{3}{3} = \mathbf{\frac{3}{6}} \qquad \frac{1}{2} \text{ x } \frac{4}{4} = \mathbf{\frac{4}{8}}$$

We won't make you count out brownies again, but we will show you what these fractions would look like as brownies. Notice that the size of the pans does not change; the only thing that does change is the number of equally-sized pieces that each pan is cut into within each pan.

$^1/_2$ $^2/_4$ $^3/_6$ $^4/_8$

Can you see that whether you get $^1/_2$, $^2/_4$, $^3/_6$, or $^4/_8$ of the pan, you still get the same amount of brownies? Do you think we could keep going with this pattern to find even more fractions equivalent to $^1/_2$?

We'll give you a hint: it has to do with the multiples of the number in the denominator. Look at the fractions under the brownie pans above. Can you see that the numbers in the denominators are multiples of two? We could write more equivalents fractions based on tenths, twelfths, fourteenths, eighteenths, and so on, using other multiples of two.

Knowing what you know about the multiples of 2, do you think there is an equivalent fraction for $^1/_2$ based on ninths?

No, because 9 is not a multiple of 2.

There's one more point you need to understand when it comes to equivalent fractions. Let's compare $^{2}/_{3}$ and $^{5}/_{9}$. Looking at the denominators we can see that 9 is a multiple of 3 (3 x 3 = 9). For these two fractions to be equivalent, the ratio that shows the relationship between the parts to the whole must be the same. In other words, both the numerator and the denominator must be multiplied by the same number, and the products must equal the numerator and denominator in the other fraction. In this case, since we multiplied the denominator by 3, three times the numerator must equal the numerator in the second fraction. 2 x 3 = 6, not five. So $^{2}/_{3}$ and $^{5}/_{9}$ are NOT equivalent fractions.

SLOW DOWN

SKILL CHECK: Find the equivalent fraction: $^{3}/_{5} = {}^{?}/_{10}$

Practice

Find the equivalent fractions.

(1) $^{1}/_{2} = {}^{?}/_{10}$ (2) $^{2}/_{3} = {}^{?}/_{12}$ (3) $^{3}/_{4} = {}^{?}/_{12}$

(4) $^{3}/_{7} = {}^{?}/_{14}$ (5) $^{4}/_{5} = {}^{?}/_{20}$ (6) $^{1}/_{9} = {}^{?}/_{36}$

Write YES if the second fraction is equivalent to the first one or NO if it is not.

(7) $^{2}/_{3} = {}^{3}/_{12}$ (8) $^{4}/_{7} = {}^{12}/_{21}$ (9) $^{2}/_{8} = {}^{4}/_{24}$ (10) $^{3}/_{10} = {}^{6}/_{30}$

Write three more equivalent fractions for the fraction given.

(11) $^{5}/_{6}$ (12) $^{3}/_{10}$ (13) $^{1}/_{6}$ (14) $^{3}/_{8}$ (15) $^{4}/_{9}$

Challenge Questions: find the equivalent fractions.

(16) $^{4}/_{8} = {}^{12}/_{?}$ (17) $^{6}/_{7} = {}^{24}/_{?}$ (18) $^{3}/_{12} = {}^{9}/_{?}$